



Financial Regime-Switching Vector
Auto-Regression
Amendment August Draft
New claims, Office Action and Response

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Chapter 1

Intro

Sir: In response to the Office action of March 4, 2010, please accept this request for continuing examination and amend the above-identified application as follows:

In the process of reviewing the thoughtful comments of the examiners, and in providing an annotation of the specification amendment to show that there was no new matter, some typos or oversights in the notation of some equations were found. These are corrected. The amendment to the specification with the changes is submitted and with annotations to show where the specification additions come from in the original specification, are inherent parts like the outside of a sphere, or are from the literature and thus are not

new matter.

Amendments to the claims begin in chapter of that title of this paper.

Responses to the examiner's comments follow.

Applicant appreciates the extensive comments of examiner and has canceled the old claims and entered new ones in accordance with examiner's suggestions.

The amendment of the specification has been retained with an extended discussion of the examiner's request for the specification amendment at the personal interview. Examiner at the personal interview asked for a tutorial level introductory explanation.

Applicant indicated he would write it on the side for the examiner. Examiner then said to put it in the specification, that he didn't want it just for him.

The introductory level of the explanation in comparison with material in finance is discussed to explain why the examiner at the personal interview would make that request for a tutorial presupposing no background. Exam-

iner wisely indicated that it should go in the specification and not be just for him.

This tutorial level explanation was desired by examiner at the personal interview to better understand and respond to the claims. This is discussed in detail why this is true in practice and why the examiner had good judgement at the personal interview in 2009 both to ask for the introductory tutorial and to insist it go in the specification and not just be for him personally on the side. No new matter was introduced by the introductory tutorial. In particular, the USPTO in its rejection based on Bansal overlooked that Bansal does not have any equities in it, so it can't be prior art to Claim 25 which had an equity as part of it. The antecedent rejections also reject items that are inherent parts, and this is pointed out later. The tutorial helps bring these inherent parts out. For example, time nodes are an inherent part of a scenario.

Applicant has repeatedly requested USPTO to draft sample valid claims for applicant as a pro se applicant. This has never been done. Applicant

requests USPTO draft valid claims based on the original application.

Until it is done, final rejection is a violation of USPTO regulations, due process of law and equal protection of the law. An agency of the government can never set aside due process of law and equal protection of law because they come from the Constitution and apply continuously.

When claims are drafted by USPTO that are valid, then final rejection is not valid, since there are valid claims. Thus final rejection is a violation of USPTO regulations, due process of law and equal protection of the law, unless there is no new matter in the entire invention and patent application. That is not the case here. USPTO's effort to try to use Bansal is in error since Bansal is for interest rates only and claim 25 was for interest rates and equity variable(s) with regime switching. Bansal was regime switching for interest rates only. The other claims also contained elements distinguishing them from Bansal. Those are based on the original specification.

http://www.uspto.gov/web/offices/pac/mpep/documents/0700_707_07_j.htm

When, during the examination of a pro se application it becomes apparent to the examiner that there is patentable subject matter disclosed in the application, the examiner should draft one or more claims for the applicant and indicate in his or her action that such claims would be allowed if incorporated in the application by amendment.

Chapter 2

Annotated Amendment to the Specification

The amendment to the specification is annotated here to show from the original specification that it is not new matter. There were typos or poor choices in some of the original notation of the Amendment to the Specification that became clear while annotating to show it is not new matter.¹ These should be changed if the amendment to the specification is accepted. These are indicated in the footnotes if not changed in the text.

¹The typos help bolster the case that the claims were not based on the tutorial added to the specification. That was for the examiner as requested. As pointed out in the response to office action portion of this document, when applicant offered to do the tutorial on the side just for the examiner, the examiner said no he didn't want it just for him but to put it in the specification. The tutorial was done at the introductory level of the personal interview, ie assuming no prior background in academic or Wall Street finance, derivative or quant programming, stochastic processes, math or physics.

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2.1 Underlining review

714 Amendments, Applicant's Action [R-6]

37 CFR 1.121 Manner of making amendments in application.

- (b) Specification. Amendments to the specification, other than the claims, computer listings (1.96) and sequence listings (1.825), must be made by adding, deleting or replacing a paragraph, by replacing a section, or by a substitute specification, in the manner specified in this section.
- (iii) The full text of any added paragraphs without any underlining; and;

2.2 Remove sentence

In paragraph [0367], please remove the sentence:

"In this case, we require that $\zeta(s, s) = 0$."

The definition is reproduced below with the sentence struck out.

2 – 1 new definition

Definition 2.1 (Continuous RS-VAR) The regime s can make a transition at any point t in continuous time. The probability (conditional on a transition occurring) of a transition from s to s' is $\zeta(s', s)$. The probability of a transition out of state s is $\eta(s)dt$. In this case, we require that $\zeta(s, s) = 0$.

We also require that the $\zeta(s', s)$ are non-negative and sum to one.

There is a vector of state variables, v , that follows the process

$$dv = (b[s] + A[s]v)dt + G[s]dw \quad (2.1)$$

where v is an n by 1 vector of variables, b is an n by 1 vector of parameters, A is n by n , G is n by k and dw is k by 1. The vector dw is a vector of Wiener processes, with mean 0 and variance dt . ♠

2.3 Additions to Specification

Please amend the specification by adding the following subsection to the specification at the end of the Best Modes chapter. This is the addition requested by the examiner in the interview in 2009. It introduces no new

matter.

3 – 1 Steps in Regime Switching Economic Scenario Generation

Further clarification of the steps in regime switching economic scenario generation is given here.

3 – 1.1 Regime Switching VAR Time Node Step

Definition 2.2 (Regime Switching Time Node Step) The steps of the regime switching time node step consist of ²

1. Determine the regime index using regime determination ³ means.
2. Use the regime index to map to the parameters for the VAR. ⁴
3. Use said mapped parameters in the VAR to generate the next economic state vector in time. ⁵

²Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

³See Chapter 9 original specification. See Hamilton [60] [61] and [62] as referenced in 2-1.

⁴See Chapter 9 original specification

⁵See Chapter 9 original specification. For example 9.1 "We have a mapping from s to b,A and G, assigning for each s, b(s), A(s) and G(s). Different values of s can then correspond to the different parameters of the univariate models for each index."

4. Use the regime index to map to auxiliary regime dependent data.⁶
5. Use said auxiliary regime dependent data with the state variable to calculate auxiliary data at the time node.⁷
6. Update the regime probability vector.⁸

An alternative order is to determine the regime index for next period at the end of the time node step. The probability vector of the regime can also be updated at the start of the time node. What is important is the sequence of time node steps, and the start and stop point of the time node can be modified slightly with no consequential change.

Definition 2.3 (Update Regime Probability State Vector) 1. Multiply the collapsed regime probability state vector by the regime probability transition matrix.⁹

⁶See Chapter 9 original specification and previous footnote.

⁷Original specification 9.2 auxiliary processing

⁸See Chapter 9 original specification, Hamilton [60], and inherent part of regime generation.

⁹Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

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Definition 2.4 (Collapse Regime Probability State Vector) ¹⁰

1. Compute the cumulative probability vector of each regime by summing from 0 to the regime index. This forms a partition of the unit interval from 0 to 1. The lower endpoint is considered part of it and the upper end point not except for the last endpoint which is part of the last partition subinterval. ¹¹
2. Generate a uniform random deviate. ¹²
3. Find the partition subinterval corresponding to this uniform random deviate. ¹³
4. Set the regime index to the index of this subinterval. ¹⁴

♠

For example, if there are two subintervals, one of probability 2/3 and one of 1/3, in the order regime 0 and regime 1, then a random deviate from 0 to 2/3 indicates regime 0 and one from 2/3 to 1 indicates regime 1. If

¹⁰Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

¹¹This is just regime simulation on its own. See Hamilton [60] [61] and [62].

¹²This is just regime simulation on its own. See Hamilton [60] [61] and [62].

¹³This is just regime simulation on its own. See Hamilton [60] [61] and [62].

¹⁴This is just regime simulation on its own. See Hamilton [60] [61] and [62].

the deviate is exactly $2/3$, then regime 1 is chosen. Obviously, this whole arrangement can be varied in many ways so as to preserve the overall effect of choosing a regime index based on the probability of the regimes.¹⁵

3 – 1.2 Regime Switching Time Node Loop

Definition 2.5 (Regime Switching Time Node Loop) Regime Switching Time Node Loop consists of the steps¹⁶

1. Regime Switching Time Node Loop Initiation Step.¹⁷
2. Iterating over a loop of regime switching time nodes.¹⁸
3. Post time node loop processing step.¹⁹

¹⁵Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

¹⁶This is partly just coining a term. See figures 1 to 4 of the original drawings.

¹⁷See figure 2 in particular.

¹⁸See figure 4 for a reference to a time node index t as an example of the iterate variable. You wouldn't have an iteration step from t to $t+1$ if you did not have a loop of time nodes t .

¹⁹This is an inherent part of a time node loop computer code. After the time loop is over, the computer has to have instructions to do something next. What it does between the time node loop in one scenario and the commencement of the next scenario with its own time node loop is the post time node loop processing step. This is an inherent part of having a time node loop inside a scenario loop. Those two loops are inherently a part of economic scenarios.

3 – 1.3 Scenario

Definition 2.6 (Scenario) ²⁰

1. Scenario initialization step ²¹
2. Time node loop ²²
3. Post time loop processing ²³

A scenario consists of a scenario initialization step, a time node loop, and a post time node processing step.

The method of generating a scenario on a computer the steps consisting of the scenario initialization step, the time node loop step, and the post time node processing step. ²⁴

²⁰See Definitions 10.11 to 10.14 from original specification, page 76. Note these are inherent parts of scenarios. The USPTO claimed Bansal as prior art, and claimed it applied to computer calculations, which is an admission of scenario and time node structure to perform the following steps. Scenario generators have existed for a long time and have the following steps as inherent parts and as industry usage. This is just terminology for the parts of scenarios and time nodes and inherent acts done at them.

²¹See the drawings and previous discussion. A scenario initialization step is an inherent part of a scenario in a computer program for scenario generation.

²²See previous discussion and drawings which manifest time nodes being iterated over.

²³See previous discussion.

²⁴See previous discussion. This sentence by itself is not anything new, since any economic scenario generator has these steps as inherent parts. This illustrates again this was a tutorial at a basic level about economic scenario generators for those with no prior background.

3 – 1.4 Scenario Initialization**Definition 2.7 (Scenario Initialization)** ²⁵

1. Reset time loop accumulators to zero.
2. Update scenario seed.

3 – 1.5 Scenario Loop**Definition 2.8 (Scenario Loop)** ²⁶

1. Scenario Loop Initialization
2. Scenario Loop
3. post scenario loop processing

²⁵See Definitions 10.11 to 10.14 from original specification, page 76. Note these are inherent parts of scenarios. The USPTO claimed Bansal as prior art, and claimed it applied to computer calculations, which is an admission of scenario and time node structure to perform the following steps. Scenario generators have existed for a long time and have the following steps as inherent parts and as industry usage. This is just terminology for the parts of scenarios and time nodes and inherent acts done at them.

²⁶See Definitions 10.11 to 10.14 from original specification, page 76. Note these are inherent parts of scenarios. The USPTO claimed Bansal as prior art, and claimed it applied to computer calculations, which is an admission of scenario and time node structure to perform the following steps. Scenario generators have existed for a long time and have the following steps as inherent parts and as industry usage. This is just terminology for the parts of scenarios and time nodes and inherent acts done at them.

3 – 1.6 Regime Switching Scenario

Definition 2.9 (Regime Switching Scenario) ²⁷

1. time initialization step
2. Regime Switching time node loop
3. post time loop processing

A regime switching scenario consists of a scenario initialization step, a regime switching time node loop, and a post time node processing step. ²⁸

The method of generating a regime switching scenario on a computer the steps consisting of the scenario initialization step, the regime switching time node loop step, and the post time node processing step. ²⁹

3 – 1.7 Regime Switching Grids

A set of grids indexed by a regime index.

²⁷Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

²⁸Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

²⁹Section 2-1 and 2-2 reference Hamilton [60] [61] and [62] and others for pure regime switching processes.

Consider the following procedure. A regime switching scenario generator that generates regime switching yield scenarios by first generating regime switching scenarios of the regime index and economic state variables and using said regime index at each time node to determine a yield grid and using the economic state variables at each time node to interpolate a yield from the yield grid so selected. This is intended for implementation on a computer in accordance with other statements in the specification.

3 – 1.8 One space variable one regime index example

The paper Tenney [120] has an extensive discussion of the Green's function numerical method including grid calculations. The Green's function in the case of one space variable and one regime index is indexed by $g[i][s'][i'][s']$.

^{30 31} Here i is the discrete index of the space variable at the previous time node, and i' at the next time node, and s is the regime index at the earlier

³⁰The use of Green's functions is indicated in the original specification. Regime switching stochastic processes are indicated in the original specification including chapter 9. The letter g is just a notation for a function, the Green's function. The indices index the discrete space variable with $[i]$ at the start of period and $[i']$ at end and regime $[s]$ at start and $[s']$ at end. Note the misprint of g should be corrected to $g[i][s][i'][s']$. This misprint did not carry over to the use of g in the equations and so had no effect in the amendment.

³¹The $g[i][s'][i'][s']$ should be changed to $g[i][s][i'][s']$.

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time node and s' the regime index at the next time node.³² Let b be the price of some security.

$$b[t][i][s] = \sum [i'][s'] g[i][s][i'][s'] b[t'][i'][s'] \quad (2.2)$$

³³ ³⁴

The regime probability transition matrix is independent of the state variable process so that the Green's function factors into the product of the transition matrix and a pure inside a regime Green's function, h ,

$$g[i][s][i'][s'] = rho[s][s'] h[s'][i][i'] \quad (2.3)$$

³⁵ Note that for pricing the transition matrix rho is risk neutral, which can

³²The use of particular letters is not introducing new matter.

³³This equation simply states that the discretized function price at one time node equals the sum of the state price times the function price at the next time node. Here $b[t][i][s]$ is at time node t say, and $b[t'][i'][s']$ at time node t' , say $t+1$. This is added for clarity in the specification. This equation is just saying there is a state price or Green's function and to discretize it. The specification states to use the Green's function method as an approximation if needed.

See 9.6 discretization in the original specification. That functions propagate this way for a Green's function when discretized follows from reference to standard texts, Beaglehole Tenney 1991, and the original specification. See also equation 1 of the GFNM 1996 paper referenced. See also section 2.2 and equation 33.

³⁴The $b[i'][s']$ should be changed to $b[t'][i'][s']$ to distinguish end of period from start of period values.

³⁵The independence is specified in some modes of the original specification, see Chapter 9 and definitions. See definition 9.3 where the state transition of the regime is done at

differ from the real probability transition matrix of the regimes. ³⁶

$$b[t][i][s] = \sum[s']rho[s][s'] \sum[i']h[s'][i][i']b[t'][i'][s'] \quad (2.4)$$

37

$$v[i][s'] = \sum[i']h[s'][i][i']b[t'][i'][s'] \quad (2.5)$$

38

$$b[i][s] = \sum[s']rho[s][s']v[i][s'] \quad (2.6)$$

39

time nodes and between time nodes we treat the process as a continuous time process without regime switching.

Note also the separation theorem for Green's functions in Beaglehole Tenney 1991. Note that in the convention here, that i' and s' are end of period.

³⁶Using risk neutral for pricing is standard and indicated in original specification.

³⁷Rewriting of above equation given the independence over time nodes specified in the specification for some modes of the invention as noted above.

³⁸Definition of notation for intermediate quantity. Note specification page 76 item 6, "Any intermeidate variable or value used or useful to calculate any of the above." is defined as a GFV.

³⁹Rewriting equation with notation.

3 – 1.9 Two space variable one regime index example

40

The Green's function in the case of two space variables and one regime index is indexed by $g[i][j][s'][i'][j'][s']$. Here i is the discrete index of the space variable at the previous time node, and i' at the next time node, and s is the regime index at the earlier time node and s' the regime index at the next time node. Let b be the price of some security.

$$b[t][i][j][s] = \text{sum } g[i][j][s'][i'][j'][s'] b[t'][i'][j'][s'] \quad (2.7)$$

The regime probability transition matrix is independent of the state variable process so that the Green's function factors into the product of the transition matrix and a pure inside a regime Green's function, h ,

$$g[i][j][s'][i'][j'][s'] = rho[s][s'] h[s'][i][j][i'][j'] \quad (2.8)$$

⁴¹ Note that for pricing the transition matrix rho is risk neutral, which can

⁴⁰The same steps are repeated with two space like indices instead of one. The references to the original specification are all the same.

⁴¹Factorization when independent is in Beaglehole Tenney 1991 paper. To have the regime prob transition matrix and state variable independent between time nodes is specified in the original specification as a mode in chapter 9.

differ from the real probability transition matrix of the regimes.⁴²

$$b[t][i][j][s] = \sum[s']rho[s][s'] \sum[i']h[s'][i][j][i'][j']b[t'][i'][j'][s'] \quad (2.9)$$

$$v[i][j][s'] = \sum[i']h[s'][i][j][i'][j']b[t'][i'][j'][s'] \quad (2.10)$$

$$b[t][i][j][s] = \sum[s']rho[s][s']v[i][j][s'] \quad (2.11)$$

The functions $h[s'][i][j][i'][j']$ may be closed form solutions for the Green's function or fundamental solution as it is sometimes called or approximations. These are outlined in the references. See in particular the paper Tenney [120] for approximation methods.⁴³

⁴²Risk neutral is used for pricing is standard.

⁴³This paper was referenced in the original specification, Tenney, Mark S. (The Green's Function Numerical Method) with the same number 120.

40CHAPTER 2. ANNOTATED AMENDMENT TO THE SPECIFICATION